BFT Embedding of Interacting Second-Class Systems*

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Abstract

The embedding procedure of Batalin, Fradkin, and Tyutin, which allows to convert a second-class system into a first-class one, is employed to convert second-class interacting models. Two cases are considered. One, is the Self-Dual model minimally coupled to a Dirac fermion field. The other, the Self-Dual model minimally coupled to a charged scalar field. In both cases, they are found equivalent interacting Maxwell-Chern-Simons type field theories. These equivalences are pushed beyond the formal level, by analysing some tree level probability amplitudes associated to the models.

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I. INTRODUCTION

It is a well estabilished fact, that the free self-dual (SD) model [1], a second-class field theory, can be viewed as a gauge fixed version of the free, first-class, Maxwel-Chern-Simons (MCS) model [2]. In a first moment, this equivalence was proved semi-classically [3] by relating the classical trajectories of the two models and, subsequently, extended to the quantum domain by comparing their Green function generating functionals and the corresponding green functions associated with them [4–6].

Although this equivalence holds even in the presence of external sources, the question of what happens if coupling to other dynamical fields are considered, is only partialy solved. Indeed, in the fermionic case, it has been shown [7] that the SD model minimally coupled to a Dirac field is equivalent to a MCS model coupled non-minimally to a magnetic current and an additional Thirring like interaction. On the other hand, the question of what happens, when coupling with complex scalar fields are considered, is much more involved and remains open. In this case, the conserved current depends explicitly on the basic fields. Therefore, approaches based on interpolating lagrangians [4,8] fail because of a non trivial matter field dependence of the corresponding functional determinant.

This paper is devoted to the study of these two cases. The main strategy adopted in this work, is to relate Green function generating functionals associated to different models in order to conclude about their equivalence. As we intend to relate second to first-class theories, the embedding procedure of Batalin, Fradkin and Tyutin (BFT) [9,10], which allows us to convert a second-class system into a first-class one, is employed. Then, an equivalent first-class version for the second-class interacting SD model can be constructed.

We start, in Sec. 2, by characterizing the second-class SD model minimally coupled to Dirac fermions on the hamiltonian level. The BFT embedding procedure is then used to generate the corresponding first-class counterpart. After verifying the existence of a unitary gauge which render the original second-class model back, we implement a particular canonical transformation which enable us to construct all the phase-space variables of the first-class

theory in terms of those of the second-class one. Despite of the nonrenormalizability of the resulting first-class model, the equivalence is tested in a tree level calculation, by comparing the probability amplitudes for the Möller scattering process in the two models.

Simillarly, in Sec. 3, the SD model coupled to a complex scalar field is considered. Once the system is characterized in the phase-space, the BFT embedding procedure is applied to generate the corresponding first-class counterpart. The existence of a unitary gauge condition secures the equivalence of the first-class and the original second-class models. Here too, a particular canonical transformation is implemented, which relate all the phase-space variables of the first-class theory in terms of those of the old second-class one. Then, we present a MCS type lagrangian density which has the same phase-space Green function generating functional realization. As in the fermionic case, this equivalence is explored by comparing the probability amplitudes for the Möller scattering process associated to the models.

The conclusions are contained in Sec. 4.

II. THE SD MODEL COUPLED TO A DIRAC FERMION FIELD

The classical and quantum dynamics of the SD model minimally coupled to a Dirac fermion field is described, in configuration space, by the lagrangian density

$$\mathcal{L}_F^{SD} = -\frac{1}{2m}\epsilon^{\mu\nu\rho}(\partial_\mu f_\nu)f_\rho + \frac{1}{2}f^\mu f_\mu + \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{D}^f_\mu \psi - M\bar{\psi}\psi \ , \tag{2.1}$$

in which $D^f_\mu = \partial_\mu - igf_\mu$ is the covariant derivative, m and M denote the boson and the fermion mass respectively and g is the coupling constant[1] . On the hamiltonian level, the system is completely characteryzed by the primary constraints

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[1]We use natural units ($c = \hbar = 1$). Our metric is $g_{00} = -g_{11} = -g_{22} = 1$. The fully antisymmetric tensor $\epsilon^{\mu\nu\rho}$ is normalized such that $\epsilon^{012} = 1$ and we define $\epsilon^{ij} = \epsilon^{0ij}$. Repeated Greek indices sum from 0 to 2, while repeated Latin indices from 1 to 2. The left-right derivative is defined by $\Phi\overleftrightarrow{D}\Psi = \Phi D\Psi - (D\Phi)\Psi$.

$$T_0^{(0)} = \pi_0 \approx 0 \ , \tag{2.2a}$$

$$T_i^{(0)} = \pi_i + \frac{1}{2m}\epsilon_{ij} f^j \approx 0 \ , i = 1, 2 \ , \tag{2.2b}$$

the secondary constraint,

$$T_3^{(0)} = \frac{1}{m}\left(f^0 - \frac{1}{m}\epsilon_{ij}\partial^i f^j\right) + \frac{g}{m}\bar\psi\gamma^0\psi \approx 0 \ , \tag{2.3}$$

and the canonical hamiltonian

$$H^{(0)} = \int d^2x \left[\frac{1}{2}f^0 f^0 + \frac{1}{2}f^i f^i - i\bar\psi\gamma^i(\partial_i - igf_i)\psi + M\,\bar\psi\psi - \ m f^0 T_3^{(0)}\right] \ . \tag{2.4}$$

In these equations, π_μ are the momentum canonical conjugate to the basic fields f^μ and the canonical Poisson bracket structure satisfyed by them imply the second-class nature of the constraints. We proceed now, defining partial Dirac δ-brackets with respect to the constrains (2.2b), in the usual manner [11]. Within the δ-bracket algebra, the constraints (2.2b) holds as strong identities and we use them to eliminate from the game the momenta π_i. Then, the only non vanishing δ-brackets are

$$\left[f^i(x^0,\vec x)\,,\, f^j(x^0,\vec y)\right]_\delta = -m\,\epsilon^{ij}\,\delta(\vec x - \vec y) \ , \tag{2.5a}$$

$$\left\{\psi_a(x^0,\vec x)\,,\, \bar\psi_b(x^0,\vec y)\,\right\}_\delta = -i\,\gamma^0_{ab}\,\delta(\vec x - \vec y) \ . \tag{2.5b}$$

The BFT embedding procedure is now applyed to the remining second-class system, defined by the constraints (2.2a) and (2.3), the Hamiltonian (2.4) and the δ-bracket algebra (2.5a). For this purpose, one starts by introducing an additional pair of canonical variables ($[\theta, \pi_\theta]_\delta = 1$), one for each second-class constraint. The new constraints and the new Hamiltonian are found, afterwards, throgh an interative scheme which, in the present case, ends after a finite number of steps. Presently, the BFT conversion procedure yields

$$T_0^{(0)} \to T_1 \ = \ \pi_0 \ + \ \theta \ \approx \ 0 \ , \tag{2.6a}$$

$$T_3^{(0)} \to T_2 \ = \ \frac{1}{m}(f^0 + \pi_\theta) - \frac{1}{m^2}\epsilon_{ij}\partial^i f^j \ + \ \frac{g}{m}\,\bar\psi\gamma^0\psi \ \approx \ 0 \ , \tag{2.6b}$$

$$H^{(0)} \to H = \int d^2x \left[\frac{1}{2}(f^0 + \pi_\theta)^2 + \frac{1}{2}(f^i + \partial^i\theta)^2 - i\bar\psi\gamma^i(\partial_i - igf_i)\psi + M\bar\psi\psi\right] \ . \tag{2.6c}$$

One can easily check that the new constraints and the new Hamiltonian are, as required, strong under involution, i.e.,

$$[\, T_A \,,\, T_B \,]_\delta \;=\; 0 \;\;, \tag{2.7a}$$

$$[\, T_A \,,\, H \,]_\delta \;=\; 0 \;\;. \tag{2.7b}$$

The converted system is, indeed, first-class and obeys an Abelian involution algebra. We construct next the unitarizing Hamiltonian (H_U) and the corresponding Green function generating functional (W_χ) [9,10]. If we denote by

$$\Psi \;\equiv\; \int d^2x \left(\bar{\mathcal{C}}_A \, \chi^A \;-\; \bar{\mathcal{P}}_A \, \lambda^A \right) \;\;, \tag{2.8}$$

$$\Omega \;\equiv\; \int d^2x \left(\bar{\pi}_A \, \mathcal{P}^A \;+\; T_A \, \mathcal{C}^A \right) \;\;, \tag{2.9}$$

the gauge fixing fermion function and the BRST charge, respectively, one has that

$$H_U \;=\; H \;-\; [\, \Psi \,,\, \Omega \,]_\delta \;\;. \tag{2.10}$$

Here, $\mathcal{C}^A$ and $\bar{\mathcal{C}}_A$ are ghost coordinates and $\bar{\mathcal{P}}_A$ and $\mathcal{P}^A$ their respective canonical conjugate momenta. Furthermore, λ^A is the Lagrange multiplier associated with the constraint T_A and $\bar{\pi}_A$ is its canonical conjugate momentum. The gauge conditions χ^A are to be chosen such that

$$\det\, [\, \chi^A \,,\, T_B \,]_\delta \;\neq\; 0 \;\;. \tag{2.11}$$

The corresponding Green function generating functional

$$W_\chi \;=\; \mathcal{N} \int [\mathcal{D}\sigma] \; \exp(i A_U) \;\;, \tag{2.12}$$

is writen in terms of the unitarizing action A_U

$$A_U = \int d^3x \left(\pi_0 \dot{f}^0 + \frac{1}{2m} f^i \epsilon_{ij} \dot{f}^j + \pi_\theta \dot{\theta} + i\bar{\psi}\gamma^0\dot{\psi} + \bar{\pi}_A \dot{\lambda}^A + \bar{\mathcal{C}}_A \dot{\mathcal{P}}^A + \bar{\mathcal{P}}_A \dot{\mathcal{C}}^A - \mathcal{H}_U \right) \;, \tag{2.13}$$

and the integration measure $[\mathcal{D}\sigma]$, involves all the variables appearing in A_U. We proceed by restricting ourselves to consider gauge conditions which do not depend upon λ^A and/or

$\bar{\pi}_A$. Then, the rescaling $\chi^A \to \chi^A/\beta$, $\bar{\pi}_A \to \beta\bar{\pi}_A$ and $\bar{\mathcal{C}}_A \to \beta\bar{\mathcal{C}}_A$ allows, at the limit $\beta \to 0$, to carry out all the integrals over the ghosts and multiplier variables [12,13], with the result

$$\mathcal{W}_\chi = \mathcal{N} \int [\mathcal{D}f^0]\, [\mathcal{D}\pi_0][\mathcal{D}f^i]\, [\mathcal{D}\pi_\theta]\, [\mathcal{D}\theta]\, [\mathcal{D}\bar{\psi}][\mathcal{D}\psi] \det[\chi^A, T_B]_\delta$$
$$\times \left(\prod_{A=1}^{2} \delta[\chi^A]\right) \left(\prod_{A=1}^{2} \delta[T_A]\right) \exp\left[i \int d^3x \left(\pi_0 \dot{f}^0 + \frac{1}{2m} f^i \epsilon_{ij} \dot{f}^j + \pi_\theta \dot{\theta} + i\bar{\psi}\gamma^0\dot{\psi} - \mathcal{H}\right)\right] . \quad (2.14)$$

Here, $\mathcal{H}$ is the Hamiltonian density corresponding to the involutive Hamiltonian (2.6c). For completeness, we mention that under an infinitesimal supertransformation generated by Ω, the reduced phase space variables present in (2.14), change as follows

$$\delta\, f^0 \;\equiv\; [\, f^0\,,\, \Omega\,]_\delta\, \varepsilon \;=\; \varepsilon^1 \;\;, \quad (2.15a)$$
$$\delta\, f^i \;\equiv\; [\, f^i\,,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{1}{m}\, \partial^i \varepsilon^2 \;\;, \quad (2.15b)$$
$$\delta\, \theta \;\equiv\; [\, \theta\,,\, \Omega\,]_\delta\, \varepsilon \;=\; \frac{1}{m} \varepsilon^2 \;\;, \quad (2.15c)$$
$$\delta\, \pi_0 \;\equiv\; [\, \pi_0\,,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{1}{m}\, \varepsilon^2 \;\;. \quad (2.15d)$$
$$\delta\, \pi_\theta \;\equiv\; [\, \pi_\theta\,,\, \Omega\,]_\delta\, \varepsilon \;=\; -\, \varepsilon^1 \;\;, \quad (2.15e)$$
$$\delta\, \psi \;\equiv\; [\, \psi\,,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{ig}{m}\, \psi\, \varepsilon^2 \;\;, \quad (2.15f)$$
$$\delta\, \bar{\psi} \;\equiv\; [\, \bar{\psi}\,,\, \Omega\,]_\delta\, \varepsilon \;=\; +\frac{ig}{m}\, \bar{\psi}\, \varepsilon^2 \;\;, \quad (2.15g)$$

where $\varepsilon^A = \mathcal{C}^A \varepsilon$ and ε is the BRST infinitesimal constant fermionic parameter. These transformation laws show that the first-class theory cannot be enterly expressed in terms of the gauge invariant combinations

$$Y^0 = f^0 + \pi_\theta \;, \quad (2.16a)$$
$$Y^i = f^i + \partial^i \theta \;. \quad (2.16b)$$

In fact, we observe that, in (2.6c), an interaction mediated by a gauge dependent f^i field is what we expect in order to preserve gauge invariance. Finally, the introduction of the gauge conditions $\chi^1 = \pi_\theta$ and $\chi^2 = \theta$, reduces the functional (2.14) to the Green function generating functional for second-class systems derived by Senjanovic [14]. They represent therefore, the unitary gauge conditions.

Most part of the investigations on BFT embedding procedure ends at this stage, by obteining the first-class counterpart of a second-class original model. We turn now to look for alternative formulations of the first-class theory. For this end, we start by decomposing the basic f^i field in canonical pairs ($[A^i, P_j]_\delta = \delta^i_j$),

$$f^i \; = \; \frac{1}{2} A^i \; + \; m\epsilon^{ij} \, P_j \;\; , \tag{2.17}$$

to write a modified generating functional,

$$\begin{aligned} \mathcal{W}_\chi \; &= \mathcal{N} \int [\mathcal{D}f^0] \, [\mathcal{D}\pi_0][\mathcal{D}A^i][\mathcal{D}P_i] \, [\mathcal{D}\theta] \, [\mathcal{D}\pi_\theta] \, [\mathcal{D}\bar{\psi}][\mathcal{D}\psi] \det[\chi^A, T_B]_\delta \\ \times &\left(\prod_{A=1}^{2} \delta[\chi^A] \right) \left(\prod_{A=1}^{2} \delta[T_A] \right) \exp\left[i \int d^3x \left(\pi_0 \dot{f}^0 + P_i \dot{A}^i + \pi_\theta \dot{\theta} + i\bar{\psi}\gamma^0\dot{\psi} - \mathcal{H} \right) \right] . \end{aligned} \tag{2.18}$$

Notice that, despite the fact that we are left with an essentially canonical phase-space, once momentum variables are reintroduced and the particular δ-brackets involved have a canonical form, the symplectic structure of the constraints (2.2b) is encoded in the composition law (2.17). To show the equivalence of the just mentioned generating functional with (2.14), we note that in expression (2.18), the Hamiltonian ($\mathcal{H}$) depends on the variables A^i and P_i only through expressions (2.17). This suggests the change of variables $A^i \to A'^i = f^i$, $P_i \to P'_i = P_i$, whose jacobian is a nonvanishing real number. Since $\mathcal{H}$ does not depend upon P'_i, the corresponding integration is straightforward and after performing it one obtains (2.14). With this, we note that in terms of the varables A^i and P_i, the first-class constraint T_2 can be rewriten as

$$mT_2 \; = \; Y^0 + \mathcal{G} - \frac{1}{2m} \epsilon_{ij} F^{ij} + g\bar{\psi}\gamma^0\psi \; \approx \; 0 \;\; , \tag{2.19}$$

where $F^{ij} = \partial^i A^j - \partial^j A^i$ while $\mathcal{G}$ has the form

$$\mathcal{G} \; = \; \partial^k P_k \; + \; \frac{1}{2m} \, \epsilon_{kl} \, \partial^k A^l \;\; . \tag{2.20}$$

We now focus on Eq.(2.18) and perform the particular canonical transformation:

$$f^0 \; \to \; A'^0 \; = \; f^0 \;\; , \tag{2.21a}$$

$$\pi_0 \;\to\; P_0' \;=\; \pi_0 \;+\; \theta \;\approx 0 \;\;, \tag{2.21b}$$

$$A^i \;\to\; A'^i \;=\; A^i \;\;, \tag{2.21c}$$

$$P_i \;\to\; P_i' \;=\; P_i \;-\; \frac{1}{m}\,\epsilon_{ij}\,\partial^j\theta \;\;, \tag{2.21d}$$

$$\theta \;\to\; \theta' \;=\; \theta \;\;, \tag{2.21e}$$

$$\pi_\theta \;\to\; \pi_\theta' \;=\; \pi_\theta \;-\; \frac{1}{2m}\,\epsilon_{ij}\,F^{ij} \;+\; f^0 \;\;, \tag{2.21f}$$

$$\psi \;\to\; \psi' \;=\; \psi \;\;, \tag{2.21g}$$

$$\bar\psi \;\to\; \bar\psi' \;=\; \bar\psi \;\;. \tag{2.21h}$$

Note that the P_0' variable is nothing but the first-class constraind T_1. Omiting the"$\prime$" superindex in the new variables and performing the π_θ integration, the generating functional turn to

$$\begin{aligned}
\mathcal{W}_\chi = \mathcal{N}\int&[\prod_{\mu=0}^{2}\mathcal{D}A^\mu][\prod_{\mu=0}^{2}\mathcal{D}P_\mu][\mathcal{D}\theta][\mathcal{D}\bar\psi][\mathcal{D}\psi]\,\det[\chi^A,T_B]_\delta\left(\prod_{A=1}^{2}\delta[\chi^A]\right)\delta[P_0]\\
\times\,\exp&\left\{i\int d^3x\left[P_\mu\dot A^\mu + \dot\theta\mathcal{G} + i\bar\psi\gamma^0\dot\psi - g\bar\psi\gamma^0\psi\dot\theta - g\bar\psi\gamma^i\psi\partial_i\theta\right.\right.\\
-&\;\frac{m^2}{2}P_iP_i + \frac{m}{2}P_i\epsilon^{ij}A^j - \frac{1}{8}A^iA^i - \frac{1}{4m^2}F^{ij}F^{ij} + \bar\psi\left(i\gamma^i\partial_i - M\right)\psi\\
-&\;\frac{g}{2}\bar\psi\gamma^i\psi A^i - mg\bar\psi\gamma^i\psi\epsilon^{ij}P_j + \frac{g}{2m}\epsilon_{ij}F^{ij}(\bar\psi\gamma^0\psi) - \frac{g^2}{2}(\bar\psi\gamma^0\psi)^2\\
-&\left.\left.\;\mathcal{G}\left(\frac{1}{2}\mathcal{G} - \frac{1}{2m}\epsilon_{ij}F^{ij} + g\bar\psi\gamma^0\psi\right)\right]\right\}\;\;,
\end{aligned}\tag{2.22}$$

where we had been made recoursivelly use of the first-class constraints T_A. The next step, is to define the gauge invariant fermions,

$$\Psi = \psi\;\exp\left(-ig\theta\right)\;\;, \tag{2.23a}$$

$$\bar\Psi = \bar\psi\;\exp\left(+ig\theta\right)\;\;, \tag{2.23b}$$

in terms of which, the generating functional is now writen,

$$\begin{aligned}
\mathcal{W}_\chi = \mathcal{N}\int&[\prod_{\mu=0}^{2}\mathcal{D}A^\mu]\,[\prod_{\mu=0}^{2}\mathcal{D}P_\mu]\,[\mathcal{D}\theta]\,[\mathcal{D}\bar\Psi][\mathcal{D}\Psi]\,\det[\chi^A,T_B]_\delta\,\delta[P_0]\\
\times&\left(\prod_{A=1}^{2}\delta[\chi^A]\right)\,\exp\left\{i\int d^3x\left[P_\mu\dot A^\mu + \dot\theta\,\mathcal{G} + i\bar\Psi\gamma^0\dot\Psi - \mathcal{K}\right]\right\}\;\;,
\end{aligned}\tag{2.24}$$

where

$$\mathcal{K} = \frac{m^2}{2}P_iP_i - \frac{m}{2}P_i\epsilon^{ij}A^j + \frac{1}{8}A^iA^i + \frac{1}{4m^2}F^{ij}F^{ij} - \bar{\Psi}\left(i\gamma^i\partial_i - M\right)\Psi$$
$$+ \frac{g}{2}\bar{\Psi}\gamma^i\Psi A^i + mg\bar{\Psi}\gamma^i\Psi\epsilon^{ij}P_j - \frac{g}{2m}\epsilon_{ij}F^{ij}(\bar{\Psi}\gamma^0\Psi) + \frac{g^2}{2}(\bar{\Psi}\gamma^0\Psi)^2$$
$$+ \mathcal{G}\left(\frac{1}{2}\mathcal{G} - \frac{1}{2m}\epsilon_{ij}F^{ij} + g\bar{\Psi}\gamma^0\Psi\right) \ , \tag{2.25}$$

is the modified Hamiltonian. Once the integration on the variable θ is carryed out, the new first-class constraint $\mathcal{G}$ can be used to suppress the terms containing it, in the above expression. Additionaly, the parcial gauge fixing $\chi^1 = A^0$ allows the P_0 and A^0 integrations. The lagrangian A^0_L variable is introduced by the Gauss-Law ($\mathcal{G}$) exponentiation. Finally, when the integrations on the momentum variables P_i are performed, we arrive at

$$\mathcal{W}_\chi = \mathcal{N}\int [\prod_{\mu=0}^{2}\mathcal{D}A^\mu]\,[\mathcal{D}\bar{\Psi}][\mathcal{D}\Psi]\,\det[\chi^2, T_2]_\delta\delta[\chi^2]\exp\left[i\int d^3x\,\mathcal{L}_F^{MCS}\right] \ , \tag{2.26}$$

where

$$\mathcal{L}_F^{MCS} = -\frac{1}{4m^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4m}\epsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho + \bar{\Psi}(i\gamma^\mu\partial_\mu - M)\Psi$$
$$+ \frac{g}{2m}\epsilon_{\mu\nu\rho}F^{\mu\nu}(\bar{\Psi}\gamma^\rho\Psi) - \frac{g^2}{2}(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi) \ . \tag{2.27}$$

The nonminimal character of the interaction in this lagrangian, is compatible with the idea of gauge invariant fermions introduced in (2.23a), and a free Gauss-Law ($\mathcal{G}$). Since $\mathcal{L}_F^{MCS}$ derives from $\mathcal{L}_F^{SD}$ through the BFT embedding procedure, they must represent equivalent descriptions of the same physical system.

To test this equivalence beyond the formal level, we proceed analysing the lowest order contribution to the electron-electron elastic scattering amplitude (Möller scattering) associated with (2.27). Within the Hamiltonian framework, the dynamics of the system is described by the canonical Hamiltonian

$$H = \int d^2z\left[\frac{m^2}{2}P_iP_i \,-\, \frac{m}{2}P_i\epsilon^{ik}A^k + \frac{1}{8}A^kA^k + \frac{1}{4m^2}F^{ij}F^{ij}\right.$$
$$- \frac{i}{2}\,\bar{\Psi}\,\gamma^k\,\cdot\partial_k\Psi \,+\, \frac{i}{2}\,(\partial_k\bar{\Psi})\,\gamma^k\,\cdot\Psi \,-\, M\,\bar{\Psi}\,\cdot\Psi \,+\, \frac{g}{2}(\bar{\Psi}\gamma^k\Psi)A^k$$
$$\left. - \frac{g}{2m}\epsilon_{ij}F^{ij}(\bar{\Psi}\gamma^0\Psi) + \frac{g^2}{2}(\bar{\Psi}\gamma^0\Psi)(\bar{\Psi}\gamma^0\Psi) - mgP_i\epsilon^{ik}(\bar{\Psi}\gamma^k\Psi)\right], \tag{2.28}$$

the primary constraint $P_0 \approx 0$, and the secondary constraint $\mathcal{G} \approx 0$. They satisfy a first-class algebra. Once the Coulomb gauge conditions are introduced $A^0 \approx 0$, $\partial^i A^i \approx 0$, the quantization is performed by means of the Dirac bracket quantization procedure [11], which states that the equal time commutation rules are abstracted by the corresponding Dirac brackets. Then the constraints, used as strong identityes, define a reduced phase-space which is expressed enterly in terms of the fermion variables Ψ and $\bar{\Psi}$, and the transversal components P_i^T, A_T^i. In terms of this variables, the only nonvanishing equal time (anti)commutators are found to be

$$\left\{\Psi(x^0,\vec{x})\,,\,\bar{\Psi}(x^0,\vec{y})\right\} = \gamma^0\,\delta(\vec{x}-\vec{y}) \;, \tag{2.29a}$$

$$\left[A_T^i(x^0,\vec{x})\,,\,P_j^T(x^0,\vec{y})\right] = i\left(\delta^{ij} - \frac{\partial_x^i\partial_x^j}{\nabla_x^2}\right)\delta(\vec{x}-\vec{y}) \;, \tag{2.29b}$$

while the Hamiltonian splits into a sum of a free part

$$H_0 = \int d^2z\left[\frac{m^2}{2}P_j^T P_j^T - \frac{m}{2}P_i^T\epsilon^{ik}A_T^k + \frac{1}{2}A_T^k A_T^k + \frac{1}{4m^2}F_T^{ij}F_T^{ij} - \bar{\Psi}\left(i\gamma^i\partial_i - M\right)\Psi\right], \tag{2.30}$$

plus an interacting part

$$H_I = \int d^2z\left[g\bar{\Psi}\cdot\gamma^i\Psi B^i + -\frac{g}{2m}\epsilon_{ij}F^{ij}(\bar{\Psi}\cdot\gamma^0\Psi) + \frac{g^2}{2}(\bar{\Psi}\cdot\gamma^0\Psi)^2\right] \;, \tag{2.31}$$

where B^i is the combination

$$B^i \;=\; A_T^i \;+\; m\epsilon^{ij}\,P_j^T \;. \tag{2.32}$$

The dot $(\cdot)$ signalizes for the symmetrysation prescription ($\psi_a\cdot\bar{\psi}_b = 1/2[\psi_a,\bar{\psi}_b]$) adopted in interaction fermion bilinears. From inspection of (2.31), it follows that the contributions of order g^2 to the above mentioned scattering amplitude, $\mathcal{R}^{(2)}$, can be grouped in five different kinds of terms,

$$\mathcal{R}^{(2)} \;=\; \sum_{a=1}^{5}\mathcal{R}_a^{(2)} \;, \tag{2.33}$$

where

$$R_1^{(2)} = \frac{ig^2}{2}(\gamma^0)_{ab}(\gamma^0)_{cd}\int d^3x \int d^3y \delta(x^0 - y^0)\delta(\vec{x} - \vec{y})$$
$$\times \langle \Phi_f | : \bar{\Psi}_a(x)\Psi_b(x)\bar{\Psi}_c(y)\Psi_d(y) : |\Phi_i\rangle \,, \tag{2.34a}$$
$$R_2^{(2)} = -\frac{g^2}{2}(\gamma^i)_{ab}(\gamma^j)_{cd}\int d^3x \int d^3y \langle \Phi_f | T \Big\{ : \bar{\Psi}_a(x)\Psi_b(x)B^i(x) :$$
$$\times : \bar{\Psi}_c(y)\Psi_d(y)B^j(y) : \Big\} |\Phi_i\rangle \,, \tag{2.34b}$$
$$R_3^{(2)} = \frac{g^2}{4m}(\gamma^i)_{ab}(\gamma^0)_{cd}\int d^3x \int d^3y \langle \Phi_f | T \Big\{ : \bar{\Psi}_a(x)\Psi_b(x)B^i(x) :$$
$$\times : \epsilon^{jk} F_T^{jk}(y)\bar{\Psi}_c(y)\Psi_d(y) : \Big\} |\Phi_i\rangle \,, \tag{2.34c}$$
$$R_4^{(2)} = \frac{g^2}{4m}(\gamma^0)_{ab}(\gamma^i)_{cd}\int d^3x \int d^3y \langle \Phi_f | T \Big\{ : \epsilon^{jk} F_T^{jk}(x)\bar{\Psi}_c(x)\Psi_d(x) :$$
$$\times : \bar{\Psi}_a(y)\Psi_b(y)B^i(y) : \Big\} |\Phi_i\rangle \,, \tag{2.34d}$$
$$R_5^{(2)} = -\frac{g^2}{8m^2}\epsilon^{kl}\epsilon^{jm}(\gamma^0)_{ab}(\gamma^0)_{cd}\int d^3x \int d^3y \langle \Phi_f | T \Big\{ : \epsilon_{kl} F^{kl}(x)\bar{\Psi}_a(x)\Psi_b(x) :$$
$$\times : \epsilon_{jm} F^{jm}(y) A_T^m(y)\bar{\Psi}_c(y)\Psi_d(y) : \Big\} |\Phi_i\rangle \,. \tag{2.34e}$$

In this equations, T is the chronological ordering product while $|\Phi_i\rangle$ and $|\Phi_f\rangle$ denote the initial and final states of the process, respectively. In the present case, they are two-electron states. Fermion states obeying the free Dirac equation in $(2+1)$-dimensions were explicitly constructed in Ref. [15], where the notation $v^{(-)}(\mathbf{p})(\bar{v}^{(+)}(\mathbf{p}))$ was employed to designate the two-component spinor describing a free electron of two-momentum $\mathbf{p}$, energy $p^0 = +(\mathbf{p}^2 + M^2)^{1/2}$, and spin $s = M/|M|$ in the initial (final) state. The plane wave expansion of the free fermionic operators Ψ and $\bar{\Psi}$ in terms of these spinors and the corresponding creation and annihilation operators goes as usual.

In terms of the initial (p_1, p_2) and final momenta (p_1', p_2'), the partial amplitudes in (2.34a) are found to read

$$R_1^{(2)} = -\frac{g^2}{2(2\pi)^2}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \Big\{ [\bar{v}^{(+)}(\vec{p}_1{}')\gamma^0 v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2{}')\gamma^0 v^{(-)}(\vec{p}_2)] \; - \; p_1' \leftrightarrow p_2' \Big\} \,, \tag{2.35a}$$
$$R_2^{(2)} = -\frac{g^2}{2(2\pi)^2}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \Big\{ [\bar{v}^{(+)}(\vec{p}_1{}')\gamma^l v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2{}')\gamma^j v^{(-)}(\vec{p}_2)]D^{lj}(k) - p_1' \leftrightarrow p_2' \Big\} \,, \tag{2.35b}$$
$$R_3^{(2)} = -\frac{g^2}{2(2\pi)^2}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$

$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')\gamma^l v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')\gamma^0 v^{(-)}(\vec{p}_2)]\Gamma^l(k) \; - \; p_1' \leftrightarrow p_2' \right\}, \tag{2.35c}$$

$$R_4^{(2)} = -\frac{g^2}{2(2\pi)^2}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')\gamma^0 v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')\gamma^l v^{(-)}(\vec{p}_2)]\Gamma^l(-k) \; - \; p_1' \leftrightarrow p_2' \right\}, \tag{2.35d}$$

$$R_5^{(2)} = -\frac{g^2}{2(2\pi)^2}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')\gamma^0 v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')\gamma^0 v^{(-)}(\vec{p}_2)]\Lambda(k) \; - \; p_1' \leftrightarrow p_2' \right\}, \tag{2.35e}$$

where

$$D^{ij}(k) = \frac{i}{k^2 - m^2 + i\varepsilon}\left(m^2\delta^{ij} + k^i k^j - im\epsilon^{ij}k^0\right), \tag{2.36a}$$

$$\Gamma^i(k) = \frac{i}{k^2 - m^2 + i\varepsilon}\left(k^i k^0 + im\epsilon^{in}k^n\right), \tag{2.36b}$$

$$\Lambda(k) = \frac{i}{k^2 - m^2 + i\varepsilon}|\mathbf{k}|^{\mathbf{2}}, \; . \tag{2.36c}$$

and

$$k \equiv p_1' - p_1 \; = \; -(p_2' - p_2), \tag{2.37}$$

is the momentum transfer. Combining the above parcial amplitudes in (2.33), the Möller scattering amplitude obtained from (2.27) is given by

$$R^{(2)} \; = \left(-\frac{g^2}{2(2\pi)^2}\right)\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')\gamma^\mu v^{(-)}(\vec{p}_1)]\, D_{\mu\nu}(k)\, [\bar{v}^{(+)}(\vec{p}_2')\gamma^\nu v^{(-)}(\vec{p}_2)] - p_1' \leftrightarrow p_2' \right\}, \tag{2.38}$$

where

$$D_{\mu\nu}(k) = -\frac{i}{k^2 - m^2 + i\varepsilon}\left(g_{\mu\nu} \; - \frac{k_\mu k_\nu}{m^2} + \, im\epsilon_{\mu\nu\rho}\frac{k^\rho}{m^2}\right) \; . \tag{2.39}$$

The above result is exactlly the one obtained for the same calculation in the SD model minimally coupled to Dirac fermions [16]. The result shows that, although the time ordered product of the free MCS field is represented by a nonlocal expression [17], the particular form of the interaction in (2.27) is enough to ensures that the effective propagator $D_{\mu\nu}$ which emerge from (2.38) is in fact local an coincides with the propagator of the free SD field.

III. THE SD MODEL COUPLED TO A CHARGED SCALAR FIELD

The dynamics of the SD model minimally coupled to a charged scalar field is described by the lagrangian density

$$\mathcal{L}_S^{SD} = -\frac{1}{2m}\epsilon^{\mu\nu\rho}(\partial_\mu f_\nu)f_\rho + \frac{1}{2}\, f^\mu f_\mu + (D^f_\mu \phi)^*(D^{f\mu}\phi) - M^2\phi^*\phi \ . \tag{3.1}$$

In phase-space, the system is characterized by the primary constraints

$$T_0^{(0)} = \pi_0 \ \approx \ 0 \ \ , \tag{3.2a}$$

$$T_i^{(0)} = \pi_i + \frac{1}{2m}\epsilon_{ij}f^j \approx 0 \ , i = 1, 2 \ \ , \tag{3.2b}$$

the secondary constraint,

$$T_3^{(0)} = \frac{1}{m}\left(f^0 - \frac{1}{m}\epsilon_{ij}\partial^i f^j\right) - \frac{ig}{m}\left(\pi_\phi\phi - \pi_{\phi^*}\phi^*\right) \approx \ 0 \ \ , \tag{3.3}$$

and the canonical hamiltonian

$$H^{(0)} = \int d^2x \, \left[\frac{1}{2}f^0 f^0 + \frac{1}{2}f^i f^i + \pi_\phi\pi_{\phi^*} + (D^f_i\phi)^*(D^f_i\phi) + M^2\phi^*\phi - \ m f^0 T_3^{(0)}\right] \ . \tag{3.4}$$

Now, we only mention the steps we done here, which are common to the previous case. First, we define Dirac δ-brackets with respect to the constraints (3.2b), so that within the δ-bracket algebra, they holds as strong identities and can be used to eliminate the momenta π_i from the game. The nonvanishing δ-brackets are:

$$\left[f^i(x^0,\vec{x})\, , \, f^j(x^0,\vec{y})\right]_\delta \ = -m\epsilon^{ij}\delta(\vec{x}-\vec{y}) \ , \tag{3.5a}$$

$$\left[\phi(x^0,\vec{x})\, , \, \pi_\phi(x^0,\vec{y})\,\right]_\delta \ = \delta(\vec{x}-\vec{y}) \ \ , \tag{3.5b}$$

$$\left[\phi^*(x^0,\vec{x}), \pi_{\phi^*}(x^0,\vec{y})\right]_\delta \ = \delta(\vec{x}-\vec{y}) \ \ . \tag{3.5c}$$

The BFT embedding procedure is then applyed to the remining system, defined by the constraints (3.2a), (3.3) and the Hamiltonian (3.4). To this end, a canonical pair of phase-space variables ($[\theta, \pi_\theta]_\delta = 1$) is introduced. In terms of them, the BFT embedding procedure gives

$$T_0^{(0)} \rightarrow T_1 \;=\; \pi_0 + \theta \approx 0 \;\;, \tag{3.6a}$$

$$T_3^{(0)} \rightarrow T_2 \;=\; \frac{1}{m}(f^0 + \pi_\theta) - \frac{1}{m^2}\epsilon_{ij}\partial^i f^j \;-\; \frac{ig}{m}(\pi_\phi \phi - \pi_{\phi^*}\phi^*) \;\approx\; 0 \;\;, \tag{3.6b}$$

$$H^{(0)} \rightarrow H = \int d^2x \left[\frac{1}{2}(f^0 + \pi_\theta)^2 + \frac{1}{2}(f^i + \partial^i\theta)^2 + (D_i^f\phi)^*(D_i^f\phi) + M^2\phi^*\phi\right] \;\;. \tag{3.6c}$$

The new constraints and the new Hamiltonian satisfy a strongllу involutive algebra, and define in this way, a first-class theory. The unitarizing Hamiltonian (H_U) and the unitarizing action (A_U) can be constructed in a form closer to that of the previous section. Once the ghost variables are integrated out, the resulting Green function generating functional has the form

$$\begin{aligned}\mathcal{W}_\chi = \mathcal{N}\int [\mathcal{D}f^0][\mathcal{D}\pi_0][\mathcal{D}f^i][\mathcal{D}\pi_\theta][\mathcal{D}\theta][\mathcal{D}\phi][\mathcal{D}\pi_\phi][\mathcal{D}\phi^*][\mathcal{D}\pi_{\phi^*}]\det[\chi^A, T_B]_\delta \left(\prod_{A=1}^{2}\delta[\chi^A]\right) \\ \times \left(\prod_{A=1}^{2}\delta[T_A]\right)\exp\left[i\int d^3x\left(\pi_0\dot{f}^0 + \frac{1}{2m}f^i\epsilon_{ij}\dot{f}^j + \pi_\theta\dot{\theta} + \pi_\phi\dot{\phi} + \pi_{\phi^*}\dot{\phi}^* - \mathcal{H}\right)\right],\end{aligned} \tag{3.7}$$

where, $\mathcal{H}$ is the Hamiltonian density corresponding to the involutive Hamiltonian (3.6c). Furthermore, under an infinitesimal supertransformation generated by the BRST charge (Ω), the phase-space variables present in (3.7), change as follows

$$\delta\, f^0 \;\equiv\; [\, f^0\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; \varepsilon^1 \;\;, \tag{3.8a}$$

$$\delta\, f^i \;\equiv\; [\, f^i\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{1}{m}\,\partial^i\varepsilon^2 \;\;, \tag{3.8b}$$

$$\delta\,\theta \;\equiv\; [\,\theta\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; \frac{1}{m}\varepsilon^2 \;\;, \tag{3.8c}$$

$$\delta\,\pi_0 \;\equiv\; [\,\pi_0\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{1}{m}\,\varepsilon^2 \;\;, \tag{3.8d}$$

$$\delta\,\pi_\theta \;\equiv\; [\,\pi_\theta\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; -\,\varepsilon^1 \;\;, \tag{3.8e}$$

$$\delta\,\phi \;\equiv\; [\,\phi\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{ig}{m}\,\phi\,\varepsilon^2 \;\;, \tag{3.8f}$$

$$\delta\,\phi^* \;\equiv\; [\,\phi^*\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; +\frac{ig}{m}\,\phi^*\,\varepsilon^2 \;\;, \tag{3.8g}$$

$$\delta\,\pi_\phi \;\equiv\; [\,\pi_\phi\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; +\frac{ig}{m}\,\pi_\phi\,\varepsilon^2 \;\;, \tag{3.8h}$$

$$\delta\,\pi_{\phi^*} \;\equiv\; [\,\pi_{\phi^*}\, ,\, \Omega\,]_\delta\, \varepsilon \;=\; -\frac{ig}{m}\,\pi_{\phi^*}\,\varepsilon^2 \;\;. \tag{3.8i}$$

The above transformation laws shows that the first-class Hamiltonian (see Eq.(3.6c)) cannot be enterly expressed in terms of the gauge invariant combinations

$$Y^0 = f^0 + \pi_\theta \ , \tag{3.9a}$$

$$Y^i = f^i + \partial^i\theta \ . \tag{3.9b}$$

The interaction is still madiated by the gauge dependent field f^i. Finally, the auxiliary conditions $\chi^1 = \pi_\theta$ and $\chi^2 = \theta$, restores the original second-class model and define therefore, the unitary gauge.

Now, as in section 2, we turn to look for alternative formulations for the first-class theory. We start by decomposing the basic f^i field in canonical pairs ($[A^i, P_j]_\delta = \delta^i_j$) as

$$f^i \ = \frac{1}{2}A^i \ + \ m\epsilon^{ij}\,P_j \ , \tag{3.10}$$

to write the modified generating functional,

$$\begin{aligned}\mathcal{W}_\chi = \mathcal{N}\int[\mathcal{D}f^0][\mathcal{D}\pi_0][\mathcal{D}A^i][\mathcal{D}P_i][\mathcal{D}\theta][\mathcal{D}\pi_\theta][\mathcal{D}\phi][\mathcal{D}\pi_\phi][\mathcal{D}\phi^*][\mathcal{D}\pi_{\phi^*}]\det[\chi^A, T_B]_\delta \\ \times\left(\prod_{A=1}^{2}\delta[\chi^A]\right)\left(\prod_{A=1}^{2}\delta[T_A]\right)\exp\left[i\int d^3x\left(\pi_0\dot{f}^0 + P_i\dot{A}^i + \pi_\theta\dot{\theta} + \pi_\phi\dot{\phi} + \pi_{\phi^*}\dot{\phi}^* - \mathcal{H}\right)\right] .\end{aligned} \tag{3.11}$$

The equivalence of the above Green function generating functional with (3.7) is secured by the same line of reasoning presented in section 2. The first-class constraint T_2, rewriten in terms of the canonical variables A^i and P_i is

$$mT_2 \ = \ Y^0 + \mathcal{G} - \frac{1}{2m}\epsilon_{ij}F^{ij} - ig(\pi_\phi\phi - \pi_{\phi^*}\phi^*) \ \approx \ 0 \ , \tag{3.12}$$

where $F^{ij} = \partial^i A^j - \partial^j A^i$ and $\mathcal{G}$ is given by

$$\mathcal{G} \ = \ \partial^k P_k \ + \ \frac{1}{2m}\,\epsilon_{kl}\,\partial^k A^l \ . \tag{3.13}$$

We turn next, to Eq.(3.11), to perform the canonical transformation:

$$f^0 \ \rightarrow \ A'^0 \ = \ f^0 \ , \tag{3.14a}$$

$$\pi_0 \ \rightarrow \ P'_0 \ = \ \pi_0 \ + \ \theta \ \approx 0 \ , \tag{3.14b}$$

$$A^i \ \rightarrow \ A'^i \ = \ A^i \ , \tag{3.14c}$$

$$P_i \ \rightarrow \ P'_i \ = \ P_i \ - \ \frac{1}{m}\,\epsilon_{ij}\,\partial^j\theta \ , \tag{3.14d}$$

$$\theta \rightarrow \theta' = \theta \ , \tag{3.14e}$$

$$\pi_\theta \rightarrow \pi'_\theta = \pi_\theta - \frac{1}{2m}\epsilon_{ij}F^{ij} + f^0 \ , \tag{3.14f}$$

$$\phi \rightarrow \phi' = \phi \ , \tag{3.14g}$$

$$\phi^* \rightarrow \phi'^* = \phi^* \ , \tag{3.14h}$$

$$\pi_\phi \rightarrow \pi'_\phi = \pi_\phi \ , \tag{3.14i}$$

$$\pi_{\phi^*} \rightarrow \pi'_{\phi^*} = \pi_{\phi^*} \ . \tag{3.14j}$$

In the following we omit the"$\prime$"on the new variables. In terms of them, the first-class quantities (3.9b) read simply $Y^i = 1/2A^i + m\epsilon^{ij}P_j$ and, after the π_θ integration, the generating functional assumes the form,

$$\begin{aligned}
\mathcal{W}_\chi = \mathcal{N}\int [\prod_{\mu=0}^{2}\mathcal{D}A^\mu][\prod_{\mu=0}^{2}\mathcal{D}P_\mu][\mathcal{D}\theta][\mathcal{D}\phi][\mathcal{D}\pi_\phi][\mathcal{D}\phi^*][\mathcal{D}\pi_{\phi^*}]\det[\chi^A, T_B]_\delta \delta[P_0] \\
\times \left(\prod_{A=1}^{2}\delta[\chi^A]\right)\exp\left\{i\int d^3x\left[P_\mu\dot{A}^\mu + \dot{\theta}\mathcal{G} + \pi_\phi\left(\dot{\phi} + ig\dot{\theta}\phi\right) + \pi_{\phi^*}\left(\dot{\phi}^* - ig\dot{\theta}\phi^*\right)\right.\right. \\
- \pi_\phi\pi_{\phi^*} - \frac{m^2}{2}P_iP_i + \frac{m}{2}P_i\epsilon^{ij}A^j - \frac{1}{8}A^iA^i - \frac{1}{4m^2}F^{ij}F^{ij} \\
- (\partial_i\phi^* + ig(Y_i - \partial_i\theta)\phi^*)(\partial_i\phi - ig(Y_i - \partial_i\theta)\phi) - \frac{ig}{2m}\epsilon_{ij}F^{ij}(\pi_\phi\phi - \pi_{\phi^*}\phi^*) \\
\left.\left. + \frac{g^2}{2}(\pi_\phi\phi - \pi_{\phi^*}\phi^*)^2 - \mathcal{G}\left(\frac{1}{2}\mathcal{G} - \frac{1}{2m}\epsilon_{ij}F^{ij} - ig(\pi_\phi\phi - \pi_{\phi^*}\phi^*)\right)\right]\right\} \ .
\end{aligned} \tag{3.15}$$

The next step, is to define the gauge invariant scalars

$$\Phi = \phi \, \exp(+ig\theta) \ , \tag{3.16a}$$

$$\Phi^* = \phi^* \, \exp(-ig\theta) \ , \tag{3.16b}$$

$$\pi_\Phi = \pi_\phi \, \exp(-ig\theta) \ , \tag{3.16c}$$

$$\pi_\Phi = \pi_\phi \, \exp(+ig\theta) \ . \tag{3.16d}$$

in terms of which all terms containing θ, exepting the one involving $\mathcal{G}$, drop out to produce, after the θ integration, the generating functional

$$\begin{aligned}
\mathcal{W}_\chi = \mathcal{N}\int [\prod_{\mu=0}^{2}\mathcal{D}A^\mu][\prod_{\mu=0}^{2}\mathcal{D}P_\mu][\mathcal{D}\theta][\mathcal{D}\phi][\mathcal{D}\pi_\phi][\mathcal{D}\phi^*][\mathcal{D}\pi_{\phi^*}]\det[\chi^A, T_B]_\delta \\
\times \delta[\mathcal{G}]\delta[P_0]\left(\prod_{A=1}^{2}\delta[\chi^A]\right)\exp\left\{i\int d^3x\left[P_\mu\dot{A}^\mu + \pi_\Phi\dot{\Phi} + \pi_{\Phi^*}\dot{\Phi}^* - \mathcal{K}\right]\right\} \ .
\end{aligned} \tag{3.17}$$

Here, $\mathcal{K}$ is the modifyed Hamiltonian

$$\mathcal{K} = \pi_\Phi\pi_{\Phi^*} + \frac{m^2\alpha}{2}P_iP_i - \frac{m\alpha}{2}P_i\epsilon^{ij}A^j + \frac{\alpha}{8}A^iA^i + \frac{1}{4m^2}F^{ij}F^{ij}$$
$$+ (\partial_i\Phi^*)(\partial_i\Phi) + M^2\Phi^*\Phi - \frac{ig}{2}(\Phi^*\overleftrightarrow{\partial}_i\Phi)A^i + imgP_i\epsilon^{ij}(\Phi^*\overleftrightarrow{\partial}_j\Phi)$$
$$+ \frac{ig}{2m}\epsilon_{ij}F^{ij}(\pi_\Phi\Phi - \pi_{\Phi^*}\Phi^*) - \frac{g^2}{2}(\pi_\Phi\Phi - \pi_{\Phi^*}\Phi^*)^2 \ , \tag{3.18}$$

and α is the factor

$$\alpha \ = \ (1 + 2g^2|\Phi|^2) \ . \tag{3.19}$$

The expression (3.17), constructed from the original second-class model through the BFT embedding procedure, the canonical transformation (3.14a) and the gauge invariant scalars (3.16a), characterizes a particular phase-space realization for the Green function generating functional associated to the first-class theory.

We show now, that the non polinomial lagrangian density

$$\mathcal{L}_S^{MCS} = -\frac{1}{4m^2\alpha}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4m}\epsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho + (\partial_\mu\Phi^*)(\partial^\mu\Phi) - M^2\Phi^*\Phi$$
$$+ \frac{ig}{2m\alpha}\epsilon^{\mu\nu\rho}F_{\mu\nu}(\Phi^*\overleftrightarrow{\partial}_\rho\Phi) + \frac{g^2}{2\alpha}(\Phi^*\overleftrightarrow{\partial}_\mu\Phi)(\Phi^*\overleftrightarrow{\partial}^\mu\Phi) \ , \tag{3.20}$$

leads to the same particular phase-space realization for the Green function generating functional. Concerning the above mentioned model, we start by defining the momentum variables

$$P_0 \approx 0 \ , \tag{3.21a}$$
$$P_i = \frac{1}{m^2\alpha}F^{0i} + \frac{1}{2m}\epsilon_{ij}A^j - \frac{ig}{m\alpha}\epsilon_{ij}(\Phi^*\overleftrightarrow{\partial}_j\Phi) \ , \tag{3.21b}$$
$$\pi_\Phi = \dot{\Phi}^* + \frac{ig}{2m\alpha}(\epsilon{\cdot}F)\Phi^* + \frac{g^2}{\alpha}(\Phi^*\overleftrightarrow{\partial}_j\Phi)^2\Phi^* \ , \tag{3.21c}$$
$$\pi_{\Phi^*} = \dot{\Phi} - \frac{ig}{2m\alpha}(\epsilon{\cdot}F)\Phi - \frac{g^2}{\alpha}(\Phi^*\overleftrightarrow{\partial}_j\Phi)^2\Phi \ , \tag{3.21d}$$

canonically conjugate to the fields A^0, A^i, Φ and Φ^* respectively. The expressions (3.21c) and (3.21d) can be combined to produce the kinetic term

$$\pi_\Phi\dot{\Phi} + \pi_{\Phi^*}\dot{\Phi}^* = 2\pi_\Phi\pi_{\Phi^*} + \frac{ig}{2m\alpha}(\epsilon{\cdot}F)(\pi_\Phi\Phi - \pi_{\Phi^*}\Phi^*)$$
$$+ \frac{g^2}{\alpha}(\Phi^*\overleftrightarrow{\partial}_0\Phi)(\pi_\Phi\Phi - \pi_{\Phi^*}\Phi^*) \ , \tag{3.22}$$

and the electric charge density

$$\pi_\Phi \Phi - \pi_{\Phi^*}\Phi^* \;=\; -\frac{1}{\alpha}\left[(\Phi^* \overleftrightarrow{\partial}_0 \Phi) - \frac{ig}{m}(\epsilon{\cdot}F)|\Phi|^2\right] \;\; . \tag{3.23}$$

The momenta, thogether with the kinetic term, are used to write the canonical Hamiltonian density

$$\mathcal{H}_c \;=\; P_\mu \dot{A}^\mu + \pi_\Phi \dot{\Phi} + \pi_{\Phi^*}\dot{\Phi}^* - \mathcal{L}_S^{MCS} \;\; , \tag{3.24}$$

as

$$\begin{aligned}\mathcal{H}_c = &-A^0\left(\partial^i P_i + \frac{1}{2m}\epsilon_{ij}\partial^i A^j\right) + \pi_\Phi \pi_{\Phi^*} + (\partial_i \Phi^*)(\partial_i \Phi) + M^2 \Phi^* \Phi \\ &+ \frac{m^2\alpha}{2}P_i P_i - \frac{m\alpha}{2}P_i \epsilon^{ik}A^k + \frac{\alpha}{2}A^k A^k + \frac{1}{4m^2\alpha}F^{ij}F^{ij}(1 - \frac{2g^2}{\alpha}|\Phi|^2) \\ &- \frac{ig}{2m\alpha}(\epsilon{\cdot}F)(\Phi^* \overleftrightarrow{\partial}_0 \Phi)(1 - \frac{2g^2}{\alpha}|\Phi|^2) - \frac{g^2}{2\alpha}(\Phi^* \overleftrightarrow{\partial}_0 \Phi)^2(1 - \frac{2g^2}{\alpha}|\Phi|^2) \\ &+ imgP_i\epsilon^{ij}(\Phi^* \overleftrightarrow{\partial}_j \Phi) - \frac{ig}{2}(\Phi^* \overleftrightarrow{\partial}_i \Phi)A^i \;, \end{aligned} \tag{3.25}$$

where $1 - 2g^2|\Phi|^2/\alpha \equiv 1/\alpha$. On the other hand, the charge density (3.23) allows us to eliminate the velocity $(\Phi^* \overleftrightarrow{\partial}_0 \Phi)$. The resulting terms can be grouped to form

$$\begin{aligned}\mathcal{H}_c = &-A^0\left(\partial^i P_i + \frac{1}{2m}\epsilon_{ij}\partial^i A^j\right) + \pi_\Phi \pi_{\Phi^*} + (\partial_i \Phi^*)(\partial_i \Phi) + M^2 \Phi^* \Phi \\ &+ \frac{m^2\alpha}{2}P_i P_i - \frac{m\alpha}{2}P_i \epsilon^{ik}A^k + \frac{\alpha}{8}A^k A^k + \frac{1}{4m^2\alpha^2}F{\cdot}F(1 + 4g^2|\Phi|^2 + 4g^4|\Phi|^4) \\ &+ \frac{ig}{2m\alpha}(\epsilon{\cdot}F)(\pi_\Phi \Phi - \pi_{\Phi^*}\Phi^*)(1 + 2g^2|\Phi|^2) - \frac{g^2}{2\alpha^2}(\pi_\Phi \Phi - \pi_{\Phi^*}\Phi^*)^2\alpha^2 \\ &+ imgP_i\epsilon^{ij}(\Phi^* \overleftrightarrow{\partial}_j \Phi) - \frac{ig}{2}(\Phi^* \overleftrightarrow{\partial}_i \Phi)A^i \;, \end{aligned} \tag{3.26}$$

which, once we realize that, $1 + 4g^2|\Phi|^2 + 4g^4|\Phi|^4 \equiv \alpha^2$, reduces to

$$\begin{aligned}\mathcal{H}_c = &-A^0\left(\partial^i P_i + \frac{1}{2m}\epsilon_{ij}\partial^i A^j\right) + \pi_\Phi \pi_{\Phi^*} + (\partial_i \Phi^*)(\partial_i \Phi) + M^2 \Phi^* \Phi \\ &+ \frac{m^2\alpha}{2}P_i P_i - \frac{m\alpha}{2}P_i \epsilon^{ik}A^k + \frac{\alpha}{8}A^k A^k + \frac{1}{4m^2}F^{ij}F^{ij} \\ &+ imgP_i\epsilon^{ij}(\Phi^* \overleftrightarrow{\partial}_j \Phi) - \frac{ig}{2}(\Phi^* \overleftrightarrow{\partial}_i \Phi)A^i \\ &+ \frac{ig}{2m}(\epsilon{\cdot}F)(\pi_\Phi \Phi - \pi_{\Phi^*}\Phi^*) - \frac{g^2}{2}(\pi_\Phi \Phi - \pi_{\Phi^*}\Phi^*)^2 \;. \end{aligned} \tag{3.27}$$

The above expression, for the canonical Hamiltonian associated to the lagrangian density (3.20), coincides with that of (3.18). Moreover, we see that the time persistance of the primary constraint (3.21a), produces the free Gauss-law (3.13). In this way, the model we just described, has the same phase-space realization for the Green function generating functional as the first-class model constructed through the BFT embedding procedure. Therefore, the configuration space verssion for the Green function generating functional corresponding to (3.17) is

$$\mathcal{W}_\chi = \mathcal{N} \int [\prod_{\mu=0}^{2} \mathcal{D}A^\mu][\mathcal{D}\Phi][\mathcal{D}\Phi^*] \det[\chi^2, \mathcal{G}]_\delta \delta[\mathcal{G}]\delta[\chi^2] \exp\left\{ i \int d^3x \, \mathcal{L}_S^{MCS} \right\} \ . \tag{3.28}$$

Since $\mathcal{L}_S^{MCS}$ derives from $\mathcal{L}_S^{SD}$ through the BFT embedding procedure, they must be equivalent descriptions of the same physical reality.

Concerning the Möller scattering amplitude associated to these models, we only mention that they coincide. The calculations lead to the following common result for the amplitude

$$\begin{aligned} R^{(2)} \ &= \ -\frac{g^2}{2(2\pi)^2}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)\frac{m^2}{\sqrt{p_1^0 p_2^0 p_1'^0 p_2'^0}} \\ &\times \left\{ (p_1^\mu + p_1'^\mu) \, D_{\mu\nu}(k) \, (p_2^\nu + p_2'^\nu) - p_1' \leftrightarrow p_2' \right\} \ , \end{aligned} \tag{3.29}$$

where

$$D_{\mu\nu}(k) = -\frac{i}{k^2 - m^2 + i\varepsilon} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2} + im\epsilon_{\mu\nu\rho}\frac{k^\rho}{m^2} \right) \ , \tag{3.30}$$

is the free SD propagator.

IV. CONCLUDING REMARKS

In the present work, we have shown through two examples of interacting field theories, that the BFT embedding procedure can be viewed as a systematics to generate a set of first-class theories equivalent to a given second-class one. Our proof of equivalence is not restricted to demonstrate the equality between two functional integrals, but also, involves an explicit construction of the phase-space variables of one model in terms of those of the other. This is the meaning of the canonical transformation implemented in each case.

It has recently appeared in the literature the BFT embedding of the non Abelian SD model [18]. The generalization for this case, of our technique mounted on canonical transformations, is currently under progress.

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REFERENCES

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[1] P. K. Townsend, K. Pilch and P. Van Nieuwenhuizen, Phys. Lett. **136B**, 38 (1984).

[2] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (NY) **140**, 372 (1982).

[3] S. Deser and R. Jackiw, Phys. Lett.**139B**, 371, (1984).

[4] R. Banerjee, H. J. Rothe, and K. D. Rothe, Phys. Rev. **D52**, 3750 (1995).

[5] R. Banerjee, H. J. Rothe, and K. D. Rothe, Phys. Rev. **D55**, 6339 (1997).

[6] M. Fleck, H. O. Girotti, Int. J. Mod. Phys. **A14**, No. 27, 4287 (1999).

[7] M. Gomes, L. C. Malacarne, A. J. da Silva, Phys. Lett.**439B**, 137, (1998).

[8] E. Fradkin, F. A. Schaposnik, Phys. Lett. **338B**, 253 (1994).

[9] I. A. Batalin and E. S. Fradkin, Nucl. Phys. **B447**, 183 (1995).

[10] I. A. Batalin and I. V. Tyutin, Int. J. Mod. Phys. **A6**, 3255 (1991).

[11] P. A. M. Dirac, *Lectures on Quantum Mechanics*, Belfer Graduate School on Science, Yeshiva University (New York, 1964).

[12] E. S. Fradkin and G. Vilkovisky, Phys. Lett. **B55**, 224 (1975).

[13] M. Henneaux, Phys. Rep. **126**, 1 (1985).

[14] P. Senjanovic, *Ann. Phys.* (N.Y.) **100**, 227 (1976).

[15] H. O. Girotti, M. Gomes, and A. J. da Silva, Phys. Lett. **274B**, (1992), 357.

[16] H. O. Girotti, Int. J. Mod. Phys. **A14**, 2495 (1998).

[17] F. P. Devecchi, M. Fleck, H. O. Girotti, M. Gomes and A. J. da Silva, *Ann. Phys.* (N.Y.) **242**, 275 (1995).

[18] Yong-Wan Kim and K. D. Rothe, Nucl. Phys. **B511**, 510 (1998).